Class X
(CBSE 2019)
Mathematics
Abroad (Set-2)

Please keep a pen and paper ready for rough work

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Question 1

For what values of k does the quadratic equation $4x^2 - 12x - k = 0$ have no real roots?

SOLUTION:

We have been given the quadratic equation:

$4x^2 - 12x - k = 0$

To have no real roots means discriminant should be less than zero.

$D = b^2 - 4ac$

$b^2 - 4ac < 0$

Plugging the values in the formula of discriminant

$(-12)^2 - 4(4)(-k) < 0$

$144 + 16k < 0$

$k < -9$

Therefore, for $k<-9$ the quadratic equation will have no real roots.

Question 2

Find the distance between the points (a, b) and ($-a$, $-b$).

SOLUTION:

Using distance formula:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Here, $x_1=a,\ y_1=b,\ x_2=-a$ and $y_2=-b$

On substituting the values in the formula we get

$$\sqrt{(-a-a)^2+(-b-b)^2}$$

$$=\sqrt{(-2a)^2+(-2b)^2}$$

$$=\sqrt{4a^2+4b^2}$$

$$=2\sqrt{a^2+b^2}$$

Therefore, the distance between $(a,\ b)$ and $(-a,-b)$ is $2\sqrt{(a)^2+(b)^2}$

Question 3

Find a rational number between $\sqrt{2}$ and $\sqrt{7}$.

OR

Write the number of zeroes in the end of a number whose prime factorization is $2^2 \times 5^3 \times 3^2 \times 17$.

SOLUTION:

We know

$\sqrt{2}=1.414$

$\sqrt{7}=1.732$

So, rational number between $\sqrt{2}$ and $\sqrt{7}$ will be 1.5 = $\frac{3}{2}$.

OR

Given prime factorisation is $2^2 \times 5^3 \times 3^2 \times 17$.

A number will have zero at the end when we have 2×5.

In $2^2 \times 5^3 \times 3^2 \times 17$ we will have 2 zeroes as $\left(2^2 \times 5^2\right) \times 5 \times 3^2 \times 17$.

Question 4

Let Δ ABC ∽ Δ DEF and their areas be respectively, 64 cm^2 and 121 cm^2. If EF = 15·4 cm, find BC.

SOLUTION:

Given: $\triangle$ ABC ~ $\triangle$ DEF

We know ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\frac{ar\triangle ABC}{ar\triangle DEF} = \left(\frac{BC}{EF}\right)^2$

$\Rightarrow \frac{64}{121} = \left(\frac{BC}{15.4}\right)^2$

$\Rightarrow \left(\frac{8}{11}\right)^2 = \left(\frac{BC}{15.4}\right)^2$

$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$

$\Rightarrow BC = \frac{8\times15.4}{11} = 11.2\ cm$

Thus, BC = 11.2 cm.

Question 5

Evaluate:

$\frac{\tan\ 65^\circ}{\cot\ 25^\circ}$

OR

Express (sin 67° + cos 75°) in terms of trigonometric ratios of the angle between 0° and 45°.

SOLUTION:

$\frac{\tan 65^\circ}{\cot 25^\circ}$

$= \frac{\tan(90^\circ - 25^\circ)}{\cot 25^\circ} \quad (\because\ \tan(90^\circ - \theta) = \cot\theta)$

$= \frac{\cot 25^\circ}{\cot 25^\circ}$

$= 1$

OR

$(\sin 67^\circ + \cos 75^\circ)$

$= (\sin(90^\circ - 23^\circ) + \cos(90^\circ - 25^\circ)) \quad (\because\ \sin(90^\circ - \theta) = \cos\theta \text{ and } \cos(90^\circ - \theta) = \sin\theta\ )$

$= (\cos 23^\circ + \sin 25^\circ)$

Question 6

Find the number of terms in the A.P. : $18, 15\frac{1}{2}, 13, \ldots, -47.$

SOLUTION:

We have been given an A.P
$18, 15\frac{1}{2}, 13, \ldots -47$
Here, $a = 18, d = 15\frac{1}{2} - 18 = \frac{-5}{2}, a_n = -47$
We will find n using
$a_n = a + (n-1)d$
Plugging the values in the formula we get:
$-47 = 18 + (n-1)\left(\frac{-5}{2}\right)$

$-47 = 18 - \frac{5}{2}n + \frac{5}{2}$

$n = 27$
Therefore, there are 27 terms in an A.P

Question 7

A bag contains 15 balls, out of which some are white and the others are black. If the probability of drawing a black ball at random from the bag is $\frac{2}{3}$, then find how many white balls are there in the bag.

SOLUTION:

Total number of balls 15
Probability of drawing a black ball at random is $\frac{2}{3}$
probability of black ball + probability of white ball = 1
Probability of white ball = 1- probability of black ball

Probability of drawing a white ball = $1 - \frac{2}{3} = \frac{1}{3}$
Therefore, number of white balls = $15 \times \frac{1}{3} = 5$

Question 8

A card is drawn at random from a pack of 52 playing cards. Find the probability of drawing a card which is neither a spade nor a king.

SOLUTION:

We have total number of cards 52
And in deck of 52 cards number of spade are 13
And number of king = 4
But, out of these 4 kings, 1 king is already included in 13 spades card.
So, we will remove all the spade and king that is $52 - (13+3) = 36$

Therefore, probability of neither a spade nor a king is $\frac{36}{52} = \frac{9}{13}$

Question 9

Find the solution of the pair of equation :

$\frac{3}{x} + \frac{8}{y} = -1;\ \frac{1}{x} - \frac{2}{y} = 2,\ x,\ y \neq 0$

OR

Find the value(s) of *k* for which the pair of equations $\begin{cases} kx + 2y = 3 \\ 3x + 6y = 10 \end{cases}$ has a unique solution.

SOLUTION:

The given equations are

$\frac{3}{x} + \frac{8}{y} = -1 \qquad \ldots\ldots(1)$

$\frac{1}{x} - \frac{2}{y} = 2 \qquad \ldots\ldots(2)$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

(1) and (2) will become

$3u + 8v = -1 \qquad \ldots\ldots(3)$

$u - 2v = 2 \qquad \ldots\ldots(4)$

Multiply (4) with 4

$4u - 8v = 8 \qquad \ldots\ldots(5)$

Adding (3) and (5) we get

$7u = 7$

$\Rightarrow u = 1$

Putting this value in (4)

$1 - 2v = 2$

$\Rightarrow v = \frac{-1}{2}$

Now

$\frac{1}{x} = u$

$\Rightarrow \frac{1}{x} = 1$

$\Rightarrow x = 1$

And

$\frac{1}{y} = v$

$\Rightarrow \frac{1}{y} = \frac{-1}{2}$

$\Rightarrow y = -2$

OR

The given equations are

$kx + 2y = 3$

$3x + 6y = 10$

For a unique solution,

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

where $a_1 = k,\ a_2 = 3,\ b_1 = 2,\ b_2 = 6$

$\frac{k}{3} \neq \frac{2}{6}$

$\Rightarrow k \neq 1$

For all values of k except 1, the given linear equations will have unique solution.

Question 10

How many multiples of 4 lie between 10 and 205 ?

OR

Determine the A.P. whose third term is 16 and 7th term exceeds the 5th by 12.

SOLUTION:

We need to find the number of multiples of 4 between 10 and 205.
So, multiples of 4 gives the sequence 12, 16, ..., 204

a = 12, d = 4 and $a_n = 204$

Using the formula $a_n = a + (n-1)d$

Plugging values in the formula we get

$204 = 12 + (n-1)4$

$204 = 12 + 4n - 4$

$4n = 196$

$n = 49$

Thus, there are 49 multiples of 4 between 10 and 205.

OR

Given: 3rd term of the AP is 16.

$a_3 = 16$

$a + (3 - 1)d = 16$

$a + 2d = 16$ .....(1)

Also, 7th term exceeds the 5th term by 12.

$a_7 - a_5 = 12$

$[a + (7 - 1)d] - [a + (5 - 1)d] = 12$

$(a + 6d) - (a + 4d) = 12$

$2d = 12$

$d = 6$

From equation (1), we obtain

$a + 2(6) = 16$

$a + 12 = 16$

$a = 4$

Therefore, A.P. will be 4, 10, 16, 22, …

Question 11

Use Euclid's division algorithm to find the HCF of 255 and 867.

SOLUTION:

The given numbers are 255 and 867.
Now 867 > 255. So, on applying Euclid's algorithm we get

$867 = 255 \times 3 + 102$

Now the remainder is not 0 so, we repeat the process again on 255 and 102

$255 = 102 \times 2 + 51$

The algorithm is applied again but this time on the numbers 102 and 51

$102 = 51 \times 2 + 0$

Thus, the HCF obtained is 51.

Question 12

The point *R* divides the line segment AB, where A(−4, 0) and B(0, 6) such that $AR = \frac{3}{4}AB.$ Find the coordinates of *R*.

SOLUTION:

We have given that R divides the line segment AB

AR+ RB= AB

$\frac{3}{4}\mathrm{AB}+\mathrm{RB}=\mathrm{AB}$

$\Rightarrow \mathrm{RB}=\frac{\mathrm{AB}}{4}$

$\Rightarrow \mathrm{AR}:\mathrm{RB}=3:1$

Using section formula:

$x=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}\right),\ y=\left(\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

$m_1=3,\ m_2=1$

$x_1=-4,\ y_1=0$

$x_2=0,\ y_2=6$

Plugging values in the formula we get

$x=\frac{3\times0+1\times(-4)}{3+1},\ y=\frac{3\times6+1\times0}{3+1}$

$x=\frac{-4}{4},\ y=\frac{18}{4}$

$\Rightarrow x=-1,\ y=\frac{9}{2}$

Therefore, the coordinates of R$\left(-1,\frac{9}{2}\right)$.

Question 13

Prove that:

(sin θ + 1 + cos θ) (sin θ - 1 + cos θ) . sec θ cosec θ = 2

OR

Prove that :

$$\sqrt{\frac{\sec\ \theta-1}{\sec\ \theta+1}}+\sqrt{\frac{\sec\ \theta+1}{\sec\ \theta-1}}=2\operatorname{cosec}\ \theta$$

SOLUTION:

$\mathrm{LHS}=(\sin\theta+1+\cos\theta)(\sin\theta-1+\cos\theta).\sec\theta\operatorname{cosec}\theta$

$=\left[\sin^2\theta-\sin\theta+\sin\theta\cos\theta+\sin\theta-1+\cos\theta+\sin\theta\cos\theta-\cos\theta+\cos^2\theta\right]\frac{1}{\cos\theta}\frac{1}{\sin\theta}\quad\left(\because\ \sec\theta=\frac{1}{\cos\theta}\text{ and }\operatorname{cosec}\theta=\frac{1}{\sin\theta}\right)$

$=[1+2\sin\theta\cos\theta-1]\frac{1}{\cos\theta}\frac{1}{\sin\theta}$

$=[2\sin\theta\cos\theta]\frac{1}{\cos\theta}\frac{1}{\sin\theta}$

$=2=\mathrm{RHS}$

Hence proved

OR

$$\sqrt{\frac{\sec\theta-1}{\sec\theta+1}}+\sqrt{\frac{\sec\theta+1}{\sec\theta-1}}=\frac{\sqrt{\sec\theta-1}}{\sqrt{\sec\theta+1}}+\frac{\sqrt{\sec\theta+1}}{\sqrt{\sec\theta-1}}$$

$$=\frac{\sqrt{\sec\theta-1}\sqrt{\sec\theta-1}+\sqrt{\sec\theta+1}\sqrt{\sec\theta+1}}{\sqrt{\sec\theta+1}\sqrt{\sec\theta-1}}$$

$$=\frac{\left(\sqrt{\sec\theta-1}\right)^2+\left(\sqrt{\sec\theta+1}\right)^2}{\sqrt{(\sec\theta-1)(\sec\theta+1)}}$$

$$=\frac{\sec\theta-1+\sec\theta+1}{\sqrt{\sec^2\theta-1}}$$

$$=\frac{2\sec\theta}{\sqrt{\tan^2\theta}}$$

$$=\frac{2\sec\theta}{\tan\theta}$$

$$=\frac{2\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$=2\frac{1}{\sin\theta}$$

$$=2\operatorname{cosec}\theta$$

Question 14

In what ratio does the point P(−4, y) divide the line segment joining the points A(−6, 10) and B(3, −8) ? Hence find the value of y.

OR

Find the value of p for which the points (−5, 1), (1, p) and (4, −2) are collinear.

SOLUTION:

Let P divides the line segment AB in the ratio $k : 1$

Using section formula

$$x = \frac{m_1x_2+m_2x_1}{m_1+m_2}, y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$$

$$\mathrm{A}\,(-6,\ 10)\ \text{and}\ \mathrm{B}\,(3,\ -8)$$

$$m_1\ :\ m_2\ =\ k\ :\ 1$$

plugging values in the formula we get

$$-4 = \frac{k\times3+1\times(-6)}{\mathrm{k}+1},\ y = \frac{k\times(-8)+1\times10}{k+1}$$

$$-4 = \frac{3k-6}{k+1},\ \mathrm{y} = \frac{-8k+10}{k+1}$$

Considering only x coordinate to find the value of k

$$-4k-4 = 3k-6$$

$$-7k = -2$$

$$k = \frac{2}{7}$$

$$k\ :\ 1 = 2\ :\ 7$$

Now, we have to find the value of y

so, we will use section formula only in y coordinate to find the value of y

$$y = \frac{2\times(-8)+7\times10}{2+7}$$

$$y = \frac{-16+70}{9}$$

$y = 6$
Therefore, P divides the line segment AB in 2 : 7 ratio
And value of y is 6.

OR

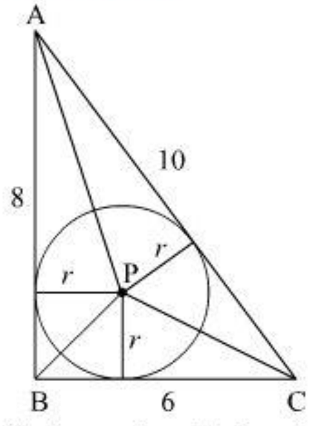
Points are collinear means the area of triangle formed by the collinear points is 0.
Using
area of triangle = $\frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1-y_2\right)\right]$
$= \frac{1}{2}[-5(p-(-2))+1(-2-1)+4(1-p)]$
$= \frac{1}{2}[-5(p+2)+1(-3)+4(1-p)]$
$= \frac{1}{2}[-5p-10-3+4-4p]$
$= \frac{1}{2}[-5p-9-4p]$

Area of triangle will be zero points being collinear
$\frac{1}{2}[-5p-4p-9]=0$
$\frac{1}{2}[-9p-9]=0$
$9p+9=0$
$p=-1$
Therefore, the value of $p = -1$.

Question 15

ABC is a right triangle in which ∠B = 90°. If AB = 8 cm and BC = 6 cm, find the diameter of the circle inscribed in the triangle.

SOLUTION:



We have given that a circle is inscribed in a triangle
Using pythagoras theorem
$(\mathrm{AC})^2=(\mathrm{AB})^2+(\mathrm{BC})^2$
$(\mathrm{AC})^2=(8)^2+(6)^2$

$(\mathrm{AC})^2 = 64 + 36$

$(\mathrm{AC})^2 = 100$

$\Rightarrow \mathrm{AC} = 10$

Area of $\triangle$ ABC $=$ area of $\triangle$ APB $+$ area of $\triangle$ BPC $+$ area of $\triangle$ APC

$\frac{1}{2} \times b \times h = \frac{1}{2} \times b_1 \times h_1 + \frac{1}{2} \times b_2 \times h_2 + \frac{1}{2} \times b_3 \times h_3$

$\frac{1}{2} \times 6 \times 8 = \frac{1}{2} \times 8 \times r + \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r$

$24 = 4r + 3r + 5r$

$24 = 12r$

$\Rightarrow r = 2$

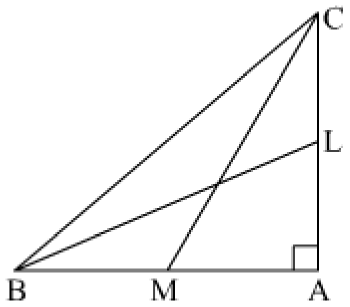
$\because\ d = 2r$

$\Rightarrow d = 2 \times 2$

$\Rightarrow d = 4$ cm

Question 16

In Figure 1, BL and CM are medians of a ΔABC right-angled at A. Prove that 4 (BL^2 + CM^2) = 5 BC^2.



OR

Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

SOLUTION:

To prove: $4\left(\mathrm{BL}^2 + \mathrm{CM}^2\right) = 5\,\mathrm{BC}^2$

Proof: In $\triangle$CAB,

Applying Pythagoras theorem,

$\mathrm{AB}^2 + \mathrm{AC}^2 = \mathrm{BC}^2$ .....(1)

In $\triangle$ABL,

$\mathrm{AL}^2 + \mathrm{AB}^2 = \mathrm{BL}^2$

$\Rightarrow \left(\frac{\mathrm{AC}}{2}\right)^2 + \mathrm{AB}^2 = \mathrm{BL}^2$

$\Rightarrow \mathrm{AC}^2 + 4\,\mathrm{AB}^2 = 4\,\mathrm{BL}^2$ .....(2)

In $\triangle$CAM,

$$CA^2 + MA^2 = CM^2$$

$$\Rightarrow \left(\frac{BA}{2}\right)^2 + CA^2 = CM^2$$

$$\Rightarrow BA^2 + 4CA^2 = 4CM^2 \qquad .....(3)$$

Adding (2) and (3)

$$AC^2 + 4AB^2 + BA^2 + 4CA^2 = 4BL^2 + 4CM^2$$

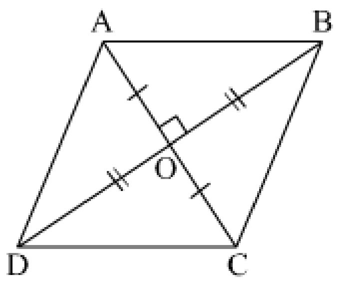
$$\Rightarrow 5AC^2 + 5AB^2 = 4\left(BL^2 + CM^2\right)$$

$$\Rightarrow 5\left(AC^2 + AB^2\right) = 4\left(BL^2 + CM^2\right)$$

$$\Rightarrow 5\left(BC^2\right) = 4\left(BL^2 + CM^2\right) \qquad (\text{From } (1))$$

Hence Proved.

OR



In ΔAOB, ΔBOC, ΔCOD, ΔAOD,

Applying Pythagoras theorem, we obtain

$$AB^2 = AO^2 + OB^2 \qquad ...(1)$$

$$BC^2 = BO^2 + OC^2 \qquad ...(2)$$

$$CD^2 = CO^2 + OD^2 \qquad ...(3)$$

$$AD^2 = AO^2 + OD^2 \qquad ...(4)$$

Adding all these equations, we obtain

$$AB^2 + BC^2 + CD^2 + AD^2 = 2\left(AO^2 + OB^2 + OC^2 + OD^2\right)$$

$$= 2\left(\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right)$$

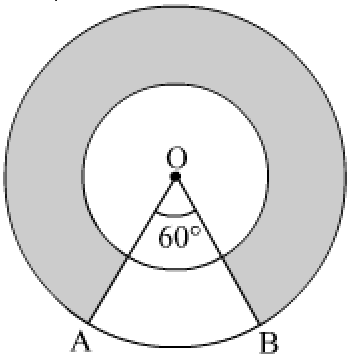
(Diagonals bisect each other)

$$= 2\left(\frac{(AC)^2}{2} + \frac{(BD)^2}{2}\right)$$

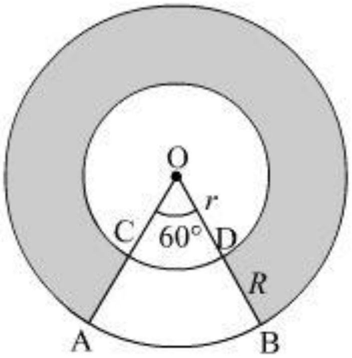
$$= (AC)^2 + (BD)^2$$

Question 17

In Figure 2, two concentric circles with centre O, have radii 21 cm and 42 cm. If ∠AOB = 60°, find the area of the shaded region.



SOLUTION:



Radius of inner circle, OC = 21 cm

Radius of outer circle, OA = 42 cm

Area of circle with radius $R = \pi R^2 = \pi(42)^2$

Area of circle with radius $r = \pi r^2 = \pi(21)^2$

Area of sector AOB$= \frac{\theta}{360} \times \pi R^2 = \frac{60}{360} \times \pi(42)^2 = \frac{\pi(42)^2}{6}$

Area of sector COD$= \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \pi(21)^2 = \frac{\pi(21)^2}{6}$

Area of shaded portion = Area of circle with radius R − Area of circle with radius *r* − [Area of sector AOB − Area of sector COD]

$= \pi(42)^2 - \pi(21)^2 - \left[\frac{\pi(42)^2}{6} - \frac{\pi(21)^2}{6}\right]$

$= \pi\left[(42)^2 - (21)^2 - \frac{1}{6}\left[(42)^2 - (21)^2\right]\right]$

$= \pi\left[\left((42)^2 - (21)^2\right)\left(1 - \frac{1}{6}\right)\right]$

$= \pi\left[(42 - 21)(42 + 21)\frac{5}{6}\right]$

$= \frac{22}{7} \times \frac{5}{6} \times 21 \times 63$

$= 3465 \text{ cm}^2$

Question 18

Calculate the mode of the following distribution :

Class :	10 − 15	15 − 20	20 − 25	25 − 30	30 − 35
Frequency :	4	7	20	8	1

SOLUTION:

Modal class is the class with highest frequency
modal class is 20 - 25
lower limit of modal class i.e l = 20
class size i.e h = 5
frequency of modal class $f_1 = 20$
frequency of preceding class $f_0 = 7$
frequency of succeeding class $f_2 = 8$
Using the formula

$$\text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Plugging the values in the formula we get

$$\text{mode} = 20 + \left(\frac{20-7}{2\times20-7-8}\right) \times 5$$

$$\text{mode} = 20 + \left(\frac{13}{25}\right) \times 5$$

$$\text{mode} = 20 + \frac{13}{5}$$

$$\text{mode} = \frac{113}{5} = 22.6$$

Question 19

A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere and hence find the surface area of this sphere.

OR

A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/hr, how much time will the tank be filled ?

SOLUTION:

A cone has been reshaped in sphere

Height of cone is 24 cm and radius of base is 6 cm

Volume of sphere = volume of cone

Volume of cone = $\frac{1}{3}\pi r^2 h$

Plugging the values in the formula we get

volume of cone = $\frac{1}{3}\pi(6)^2 24$

$$= 288\pi\ \text{cm}^3$$

Let the radius of sphere be r

Volume of sphere = $\frac{4}{3}\pi r^3$

Since, volume of cone = volume of sphere

Volume of sphere = $288\pi\ \text{cm}^3$

So,

$288\pi = \frac{4}{3}\pi r^3$

$\Rightarrow 288 = \frac{4}{3}r^3$

$\Rightarrow r^3 = 216$

$\Rightarrow r = 6\ \text{cm}$

Hence, radius of reshaped sphere is 6 cm

Now, surface area of sphere = $4\pi r^2$

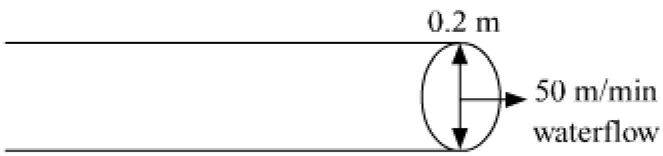
$= 4\pi(6)^2$

$= 144 \times \frac{22}{7}$

$= 452.5\ \text{cm}^2$

Therefore, surface area of sphere is $452.57\ \text{cm}^2$.

OR



Consider an area of cross-section of pipe as shown in the figure.

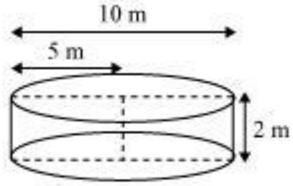
Radius (r_1) of circular end of pipe = $\frac{20}{200} = 0.1\,\text{m}$

Area of cross-section = $\pi \times r_1^2 = \pi \times (0.1)^2 = 0.01\pi\ \text{m}^2$

Speed of water = 3 km/h = $\frac{3000}{60} = 50$ metre/min

Volume of water that flows in 1 minute from pipe = 50 × 0.01π = 0.5π m^3

Volume of water that flows in t minutes from pipe = t × 0.5π m^3



Radius (r_2) of circular end of cylindrical tank = $\frac{10}{2} = 5$ m

Depth (h_2) of cylindrical tank = 2 m

Let the tank be filled completely in t minutes.

Volume of water filled in tank in t minutes is equal to the volume of water flowed in t minutes from the pipe.

Volume of water that flows in t minutes from pipe = Volume of water in tank

$t \times 0.5\pi = \pi \times (r_2)^2 \times h_2$

$t \times 0.5 = 5^2 \times 2$

$t = 100$

Therefore, the cylindrical tank will be filled in 100 minutes.

Question 20

Prove that $2+3\sqrt{3}$ is an irrational number when it is given that $\sqrt{3}$ is an irrational number.

SOLUTION:

To prove: $2+3\sqrt{3}$ is irrational, let us assume that $2+3\sqrt{3}$ is rational.
$2+3\sqrt{3}=\frac{a}{b};\ b\neq 0$ and *a* and *b* are integers.

$\Rightarrow 2b+3\sqrt{3}b=a$

$\Rightarrow 3\sqrt{3}b=a-2b$

$\Rightarrow \sqrt{3}=\frac{a-2b}{3b}$

Since *a* and *b* are integers so, $a-2b$ will also be an integer.
So, $\frac{a-2b}{3b}$ will be rational which means $\sqrt{3}$ is also rational.
But we know $\sqrt{3}$ is irrational(given).
Thus, a contradiction has risen because of incorrect assumption.
Thus, $2+3\sqrt{3}$ is irrational.

Question 21

Sum of the areas of two squares is 157 m^2. If the sum of their perimeters is 68 m, find the sides of the two squares.

SOLUTION:

Let the side of one square be *x*
And side of other square be *y*
Sum of area of two square is 157
Equation becomes

$$x^2+y^2=157 \quad \ldots\ldots(1) \quad (\because \text{ area of square is side}^2)$$

Now, sum of their perimeters is 68
Equation becomes

$4x+4y=68 \quad (\because \text{ perimeter of square is } 4\times\text{side})$

solving the two equation by substitution method

$4x+4y=68$

$x+y=17$

$\Rightarrow x=17-y \quad \ldots\ldots(2)$

Substitute (2) in (1)

$(17-y)^2+y^2=157$

$289+y^2-34y+y^2=157$

$2y^2-34y+132=0$

$y^2 - 17y + 66 = 0$

Using $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Plugging the values in the formula we get

$y = \frac{17 \pm \sqrt{289 - 4(66)}}{2}$

$y = \frac{17 \pm \sqrt{25}}{2}$

$y = \frac{17 \pm 5}{2}$

$y = \frac{12}{2}, \frac{22}{2}$

$y = 6, 11$

when $y = 6$ then $x = 11$

And when $y = 11$ then $x = 6$

Therefore, the sides of square are 6 m and 11 m.

Question 22

Find the quadratic polynomial, sum and product of whose zeroes are −1 and −20 respectively. Also find the zeroes of the polynomial so obtained.

SOLUTION:

We have been given the sum of zeroes and product of zeroes

Let us consider the general polynomial

$p(x) = ax^2 + bx + c$

Sum of zeroes is $\frac{-b}{a}$

And product of zeroes is $\frac{c}{a}$

According to question

$\frac{-b}{a} = -1 \text{ and } \frac{c}{a} = -20$

Assuming $a = 1$

$-b = -1$

$\Rightarrow b = 1$

And $c = -20$

So, the polynomial so formed is $p(x) = x^2 + x - 20$

To find the zeroes of the polynomial equate polynomial to zero.

$x^2 + x - 20 = 0$

$x^2 + 5x - 4x - 20 = 0$

$x(x+5) - 4(x+5) = 0$

$(x+5)(x-4) = 0$

$\Rightarrow x = -5, 4$

Therefore, zeroes of the polynomial are -5 and 4.

Question 23

A plane left 30 minutes later than the scheduled time and in order to reach its destination 1500 km away on time, it has to increase its speed by 250 km/hr from its usual speed. Find the usual speed of the plane.

OR

Find the dimensions of a rectangular park whose perimeter is 60 m and area 200 m^2.

SOLUTION:

Let the usual speed of the plane be *x* km/hr

And the new speed of the plane after increased by 250 is $(x+250)\ \mathrm{km}/\mathrm{hr}$

According to question

$$\frac{1500}{x}-\frac{1500}{(x+250)}=\frac{30}{60}$$

$$\Rightarrow \frac{1500x+1500\times250-1500x}{x(x+250)}=\frac{1}{2}$$

$$\Rightarrow 1500\times250\times2=x\,(x+250)$$

$$\Rightarrow 750000=x^2+250x$$

$$\Rightarrow x^2+1000x-750x-750000=0$$

$$\Rightarrow (x+1000)\,(x-750)=0$$

$$x=750,-1000$$

Speed can not be negative so -1000 will be neglected

Therefore, usual speed of the plane is 750 km/hr.

OR

Let the length of rectangle be *x*

And breadth of rectangle be *y*

xy = 200 $(\because \text{area} = \text{length}\times\text{breadth})$

And $2\,(x+y)=60$ $[\because\ \text{perimeter}=2(\text{length}+\text{breadth})]$

$\text{substitute } y=\frac{200}{x}\ \text{ in }\ 2\,(\mathrm{x}+\mathrm{y})=60$

Equation becomes:

$$2\left(x+\frac{200}{x}\right)=60$$

$$2\left(\frac{x^2+200}{x}\right)=60$$

$$2x^2-60x+400=0$$

$$x^2-30x+200=0$$

Using

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Plugging the values we get:

$x = \frac{30 \pm \sqrt{(-30)^2 - 4(1)(200)}}{2}$

$x = \frac{30 \pm 10}{2}$

$x = \frac{40}{2}, \frac{20}{2}$

$x = 20, 10$

when $x = 20$ then $y = 10$

And when $x = 10$ then $y = 20$.

Question 24

Find the value of x, when in the A.P. given below

$2 + 6 + 10 + \ldots + x = 1800$.

SOLUTION:

We have been given an A.P

$2+6+10+\ldots+x=1800$

$a = 2, d = 6 - 2 = 4, a_n = x$ and $\mathrm{s_n} = 1800$

Firstly, we will find using

$S_n = \frac{n}{2}[2a + (n-1)d]$

$1800 = \frac{n}{2}[2 \times 2 + (n-1)4]$

$1800 = \frac{4n + 4n^2 - 4n}{2}$

$900 = n^2$

$\Rightarrow n = \pm 30$

Number of terms can not be negative

$n = 30$

Now for value of x which is a_n

$a_n = a + (n-1)d$

$x = 2 + (30-1)4$

$x = 2 + 116$

$x = 118$

Therefore, value of x is 118.

Question 25

If sec θ + tan θ = m, show that $\frac{m^2-1}{m^2+1} = \sin\theta$.

SOLUTION:

$\frac{m^2-1}{m^2+1}$

$\Rightarrow \frac{(\sec\theta+\tan\theta)^2-(\sec^2\theta-\tan^2\theta)}{(\sec\theta+\tan\theta)^2+(\sec^2\theta-\tan^2\theta)}$

$\Rightarrow \frac{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta-\sec^2\theta+\tan^2\theta}{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta+\sec^2\theta-\tan^2\theta}$

$\Rightarrow \frac{2\tan\theta(\tan\theta+\sec\theta)}{2\sec\theta(\tan\theta+\sec\theta)}$

$\Rightarrow \frac{\tan\theta}{\sec\theta} = \frac{\sin\theta}{\cos\theta\sec\theta}$ $\qquad \left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

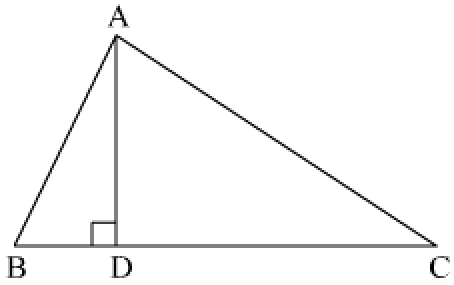
$\Rightarrow \frac{\sin\theta}{\cos\theta\times\frac{1}{\cos\theta}} = \sin\theta$ $\qquad \left(\because \cos\theta = \frac{1}{\sec\theta}\right)$

Hence, proved

Question 26

In $\triangle$ ABC (Figure 3), AD $\perp$ BC. Prove that

$AC^2 = AB^2 + BC^2 - 2BC \times BD$



SOLUTION:

Applying Pythagoras theorem in ΔADB, we obtain

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD^2 = AB^2 - DB^2$ .....(1)

Applying Pythagoras theorem in ΔADC, we obtain

$AD^2 + DC^2 = AC^2$

$AB^2 - BD^2 + DC^2 = AC^2$ [Using equation (1)]

$AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD$

$AC^2 = AB^2 + BC^2 - 2BC \times BD$

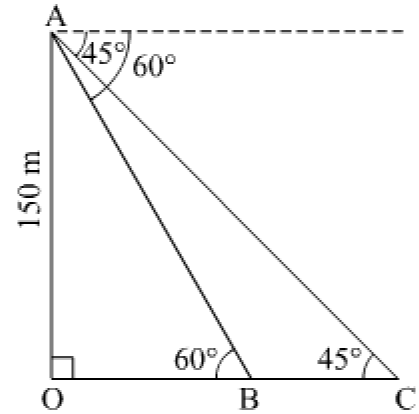
Question 27

A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/min.

OR

There are two poles, one each on either bank of a river just opposite to each other. One pole is 60 m high. From the top of this pole, the angle of depression of the top and foot of the other pole are 30° and 60° respectively. Find the width of the river and height of the other pole.

SOLUTION:



Let AO be the cliff of height 150 m.
Let the speed of boat be *x* metres per minute.
And BC be the distance which man travelled.

So, $\mathrm{BC} = 2x \quad [\because \text{Distance} = \text{Speed} \times \text{Time}]$

$\tan(60^\circ) = \frac{\mathrm{AO}}{\mathrm{OB}}$

$\sqrt{3} = \frac{150}{\mathrm{OB}}$

$\Rightarrow \mathrm{OB} = \frac{150\sqrt{3}}{3} = 50\sqrt{3}$

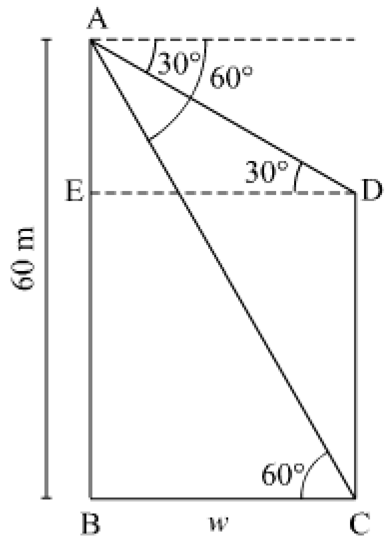
$\tan(45^\circ) = \frac{\mathrm{AO}}{\mathrm{OC}}$

$\Rightarrow 1 = \frac{150}{\mathrm{OC}}$

$\Rightarrow \mathrm{OC} = 150$
Now OC = OB + BC
$\Rightarrow 150 = 50\sqrt{3} + 2x$
$\Rightarrow x = \frac{150 - 50\sqrt{3}}{2}$
$\Rightarrow x = 75 - 25\sqrt{3}$

Using $\sqrt{3} = 1.73$
$x = 75 - 25 \times 1.732 \approx 32 \ \mathrm{m/min}$
Hence, the speed of the boat is 32 metres per minute.

OR



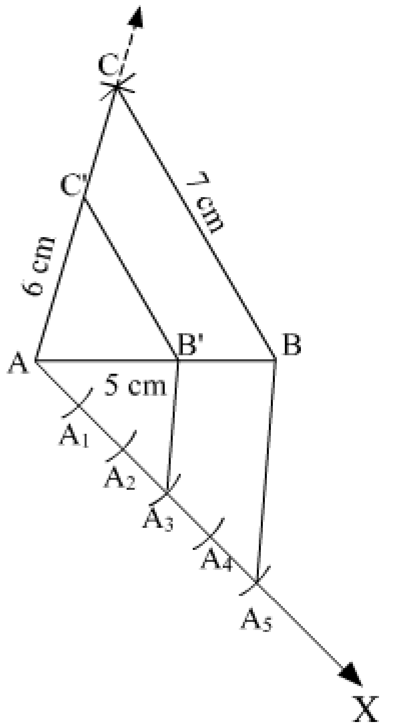
Let the width of the river be *w*.
In $\triangle$ABC,
$\tan 60^\circ = \frac{\mathrm{AB}}{\mathrm{BC}}$
$\Rightarrow \sqrt{3} = \frac{60}{w}$
$\Rightarrow w = \frac{60}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$
In $\triangle$AED,
$\tan 30^\circ = \frac{\mathrm{AE}}{\mathrm{ED}}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\mathrm{AE}}{w}$
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\mathrm{AE}}{20\sqrt{3}}$

$\Rightarrow \mathrm{AE} = 20$
Height of pole CD = AB – AE
$= 60 - 20 = 40 \text{ m}$
Thus, width of river is $20\sqrt{3} = 20 \times 1.732 = 34.64 \text{ m}$
Height of pole = 40 m

Question 28

Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of the first triangle.

SOLUTION:



1. Draw a line AB = 5 cm and draw a ray from A and taking A as centre cut an arc at C of 6 cm and taking B as centre cut an arc of 7 cm at C
2. Draw AX such that ∠BAX is an acute angle.
3. Cut 5 equal arcs AA_1, A_1A_2, A_2A_3, A_3A_4 and A_4A_5.
4. Join A_5 to B and draw a line through A_3 parallel to A_5B which meets AB at B'.

   Here, AB' = $\frac{3}{5}$ AB

5. Now draw a line through B' parallel to BC which joins AC at C'.

Here, B'C' = $\frac{3}{5}$BC and AC'=$\frac{3}{5}$ AC

Thus, AB'C' is the required triangle.

Question 29

Calculate the mean of the following frequency distribution:

Class:	10−30	30−50	50−70	70−90	90−110	110−130
Frequency:	5	8	12	20	3	2

OR

The following table gives production yield in kg per hectare of wheat of 100 farms of a village:

Production yield (kg/hectare):	40−45	45−50	50−55	55−60	60−65	65−70
Number of farms	4	6	16	20	30	24

Change the distribution to a 'more than type' distribution, and draw its ogive.

SOLUTION:

Class	frequency (f_i)	Class mark (x_i)	$f_i x_i$
10-30	5	$\frac{10+30}{2}=20$	100
30-50	8	$\frac{30+50}{2}=40$	320
50-70	12	$\frac{50+70}{2}=60$	720
70-90	20	$\frac{70+90}{2}=80$	1600
90-110	3	$\frac{90+110}{2}=100$	300
110-130	2	$\frac{110+130}{2}=120$	240
	$\sum f_i=50$		$\sum f_i x_i=3280$

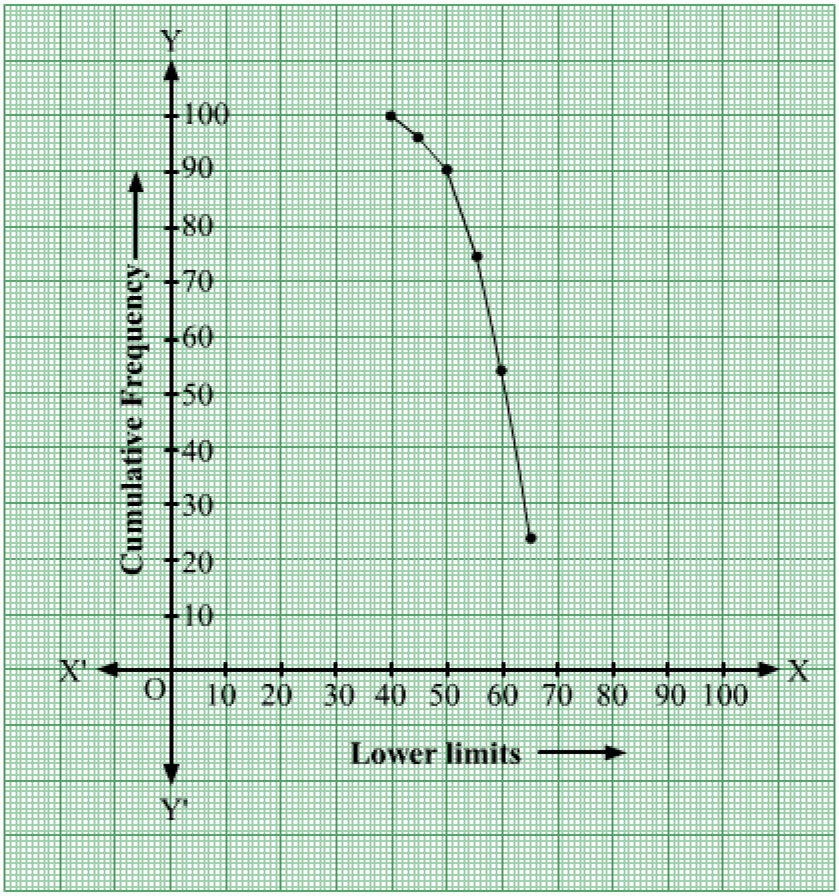
Using: mean = $\frac{\sum f_i x_i}{\sum f_i}$

substituting the values in the formula

mean$=\frac{3280}{50}=65.6$

OR

Production yield	Cumulative frequency
more than 40	100
more than 45	96
more than 50	90
more than 55	74
more than 60	54
more than 65	24



Question 30

A container opened at the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container, at the rate of ₹ 50 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 10 per 100 cm^2. (Take π = 3·14)

SOLUTION:

We have to find the cost of milk which can completely fill the container
Volume of container = Volume of frustum

$= \frac{1}{3}\pi h\left(r_1{}^2 + r_2{}^2 + r_1 r_2\right)$

Here,

height $= 16$ cm

radius of upper end $= 20$ cm

And radius of lower end $= 8$ cm

Plugging the values in the formula we get

$$\text{Volume of container} = \frac{1}{3} \times 3.14 \times 16 \left((20)^2 + (8)^2 + 20 \times 8 \right)$$
$$= \frac{1}{3} \times 50.24 (400 + 64 + 160)$$
$$= \frac{1}{3} \times 50.24 (624)$$
$$= 10449.92 \ \text{cm}^3$$
$$= 10.449 \text{ litre} \qquad (\because 1 \text{ litre} = 1000 \ \text{cm}^3)$$

Cost of 1 litre milk is Rs 50
Cost of 10.449 litre milk = 50 × 10.449 = Rs 522.45
We will find the cost of metal sheet to make the container
Firstly, we will find the area of container
Area of container = Curved surface area of the frustum + area of bottom circle (∵ container is closed from bottom)
Area of container = $\pi (r_1 + r_2) l + \pi r^2$
Now, we will find l

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$
$$l = \sqrt{(16)^2 + (20 - 8)^2}$$
$$l = \sqrt{(16)^2 + (12)^2}$$
$$l = \sqrt{256 + 144}$$
$$l = \sqrt{400}$$
$$l = 20 \ \text{cm}$$

$$\text{Area of frustum} = 3.14 \times 20 (20 + 8)$$
$$= 1758.4 \ \text{cm}^2$$

$$\text{Area of bottom circle} = 3.14 \times 8^2 = 200.96 \ \text{cm}^2$$
$$\text{Area of container} = 1758.4 + 200.96$$
$$= 1959.36 \ \text{cm}^2$$

Cost of making $100 \ \text{cm}^2 = \text{Rs } 10$
Cost of making $1 \ \text{cm}^2 = \frac{10}{100} = \text{Rs } \frac{1}{10}$
Cost of making $1959.36 \ \text{cm}^2 = \frac{1}{10} \times 1959.36 = 195.936$

Hence, cost of milk is Rs 522.45
And cost of metal sheet is Rs 195.936